rate at which the planet would lose its atmosphere, since it takes no account of molecules which describe free paths beyond the limit and fall back again. To further exhibit the results in a tangible form, the rate of flow is estimated by the number of years in which the total amount of gas escaping across the critical surface would be equal to the amount of the gas in a layer covering the surface of the planet to the depth of 1 cm. This measure is independent of the actual quantity of the gas under consideration existing in the atmosphere, since, if this quantity be increased, the rate of flow across the critical surface and the amount of gas present in the surface layer 1 cm. thick will be increased in the same proportion.

If a gas of molecular weight 2, such as helium, be supposed to exist in the earth's atmosphere, the loss in question would occupy $3{\cdot}5 \times 10^{36}$ years at $-73°$ C., 3×10^{19} years at 27°, $8{\cdot}4 \times 10^{10}$ years at 127° C., 6×10^{5} years at 227° C., and 222 years at 327° C.

If we halve the absolute temperatures we have the conditions applicable to hydrogen, the losses in question therefore taking place in $8{\cdot}4 \times 10^{10}$ years at $-73°$ C., 6×10^{5} years at $-23°$ C., and 222 years at 27° C.

For water vapour on Mars, the corresponding results are $1{\cdot}2 \times 10^{33}$ years at $-73°$, $1{\cdot}9 \times 10^{16}$ years at 27°, $2{\cdot}4 \times 10^{9}$ years at 127°, $4{\cdot}3 \times 10^{5}$ years at 227° and 106 years at 327°.

These figures indicate that helium cannot practically escape from our atmosphere at existing temperatures, nor can vapour of water escape from the atmosphere of Mars. A leakage may and undoubtedly does take place which may appear considerable when estimated by the number of actual molecules escaping, but it is wholly inappreciable relative to the mass of gas left behind.

At a future time I propose to examine the corresponding results, based on the hypothesis that the atmosphere of a planet is distributed according to the adiabatic instead of the isothermal law.

"Combinatorial Analysis.—The Foundations of a New Theory."

By Major P. A. MacMahon, R.A., D.Sc., F.R.S. Received March 19,—Read April 5, 1900.

(Abstract.)

The object of the paper is to exhibit the processes of the infinitesimal calculus and of the calculus of finite differences as combinatorial processes. A large class of problems can be dealt with by designing on the one hand a function, and on the other hand an operation, in such wise that when the operation is performed upon the function a

number results which enumerates the combinations with which the problem is concerned. The problems to which the method is applicable are those which are, directly or indirectly, associated with lattices, and it is remarkable that, for the most part, they are such as have been hitherto regarded as unassailable by the processes of pure mathematics.

If a problem be presented for solution, it may be a difficult matter to design the function and the operation, the combination of which furnishes the solution. The plan here adopted is to consider certain functions and operations, and then to inquire into the nature of the associated problems. The author has attempted to place the method on a sure foundation, to give illustrative examples of gradually increasing complexity, and to indicate some promising lines of future investigation. The only published work connected with the subject is the author's paper, entitled "A New Method in Combinatory Analysis, with application to Latin Squares and Associated Questions," which will be found in the 'Trans. Camb. Phil. Soc.,' vol. xvi, Part IV, p. 262.

"Über Reihen auf der Convergenzgrenze." Von EMANUEL LASKER, Dr. Philos. Communicated by Major MACMAHON, F.R.S. Received March 15,—Read April 5, 1900.

(Abstract.)

The essay is divided into three chapters.

A limit operation of any kind, for instance, simple or multiple integration; the formation of infinite sums or products; or any combination of these operations gives rise to certain typical considerations which form the subject of the first chapter. To fix the ideas, let

$$F(x) = u_1 + u_2 + \ldots\ldots + u_n + \ldots\ldots,$$

where the u_i are analytical functions of x; and let $x = \xi$ be a point on the curve which forms the boundary of the region of convergence of the series. Let C be a curve within the region of convergence terminating in $x = \xi$. In that case $F(\xi)$ is generally different from

$$\lim. F(x),$$
$$(\lim. x = \xi),$$

x varying on the curve C. It is a matter of the greatest importance to examine the relations between these two mathematical conceptions. If $u_1 + u_2 + \ldots\ldots + u_n + \ldots\ldots$ converges where $x = \xi$, $F(\xi)$ has a definite